1. ZT, Varianta B

(1.) Označme jevy: A ... dítě má alergii

B_1 ... oba rodiče jsou alergici

B_2 ... jeden rodič je alergik

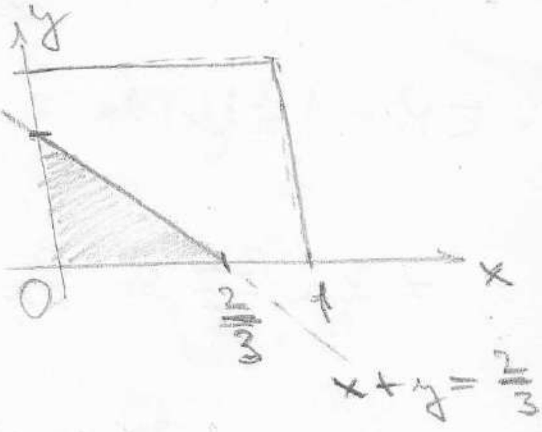
B_3 ... žádný rodič není alergik

$$P(A) = \sum_{i=1}^{3} P(A|B_i)\cdot P(B_i) = 0{,}7\cdot 0{,}05 + 0{,}3\cdot 0{,}2 + 0{,}1\cdot 0{,}75$$

$$= 0{,}035 + 0{,}06 + 0{,}075 = \underline{0{,}17}$$

(2.) Náhodně vybereme dvě čísla $x, y \in (0,1)$, jaká je pravděpodobnost, že $x+y < \frac{2}{3}$?

$$P[x+y<\tfrac{2}{3}] = \frac{\text{obsah trojúhelníku}}{\text{obsah čtverce}} = \frac{\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3}}{1} = \underline{\underline{\frac{2}{9}}}$$



(4.) 10 žárovek, z nich 2 vadné

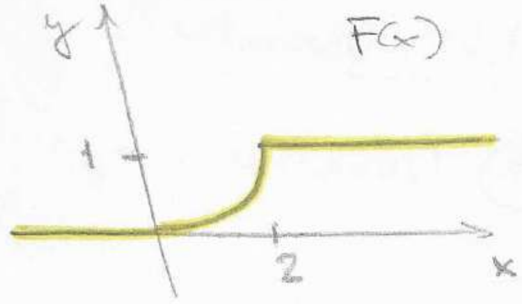
X ... počet vadných žárovek ze 3 náhodně vybraných

a) $$P[X=1] = \frac{\binom{2}{1}\cdot\binom{8}{2}}{\binom{10}{3}} = \frac{2\cdot\frac{8!}{6!\cdot 2!}}{\frac{10!}{3!\cdot 7!}} = \frac{\frac{8\cdot 7\cdot \cancel{6!}}{\cancel{6!}}}{\frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{6\cdot \cancel{7!}}} = \frac{6\cdot 7}{10\cdot 9} = \underline{\underline{\frac{7}{15}}}$$

b) $$P[X=k] = \frac{\binom{2}{k}\cdot\binom{8}{3-k}}{\binom{10}{3}}, \quad k\in\{0,1,2,3\}$$

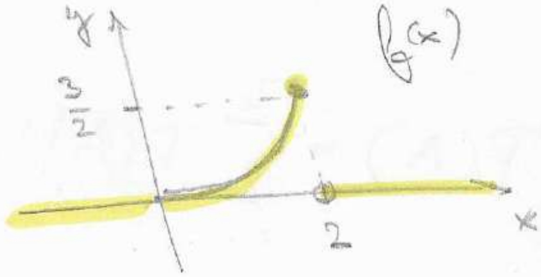
hypergeometrické rozdělení

(5.) $F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{8}x^3, & x \in [0,2] \\ 1, & x > 2 \end{cases}$



hustota:

$$f(x) = \begin{cases} \frac{3x^2}{8}, & x \in [0,2] \\ 0, & x \notin [0,2] \end{cases}$$



- $EX = \int_{\mathbb{R}} x f(x)\, dx = \int_0^2 x \cdot \frac{3x^2}{8}\, dx = \frac{3}{8} \int_0^2 x^3\, dx = \frac{3}{8} \cdot \left[\frac{x^4}{4}\right]_0^2 =$

$$= \frac{3}{8} \cdot \frac{16}{4} = \underline{\underline{\frac{3}{2}}}$$

- $EX^2 = \int_{\mathbb{R}} x^2 f(x)\, dx = \int_0^2 x^2 \cdot \frac{3x^2}{8}\, dx = \frac{3}{8} \int_0^2 x^4\, dx = \frac{3}{8} \cdot \left[\frac{x^5}{5}\right]_0^2$

$$= \frac{3}{8} \cdot \frac{32}{5} = \frac{12}{5}$$

- $\operatorname{var} X = EX^2 - (EX)^2 = \frac{12}{5} - \left(\frac{3}{2}\right)^2 = \frac{12}{5} - \frac{9}{4} = \frac{48-45}{20}$

$$= \underline{\underline{\frac{3}{20}}}$$

- $P\left[1 \leq X < \frac{3}{2}\right] = F\left(\frac{3}{2}\right) - F(1) = \frac{1}{8} \cdot \left(\frac{3}{2}\right)^3 - \frac{1}{8} \cdot 1^3 =$

$$= \frac{27}{64} - \frac{1}{8} = \frac{27-8}{64} = \underline{\underline{\frac{19}{64}}}$$

(6.)

$$f_X(x) = \begin{cases} 12x(x-1)^2 & , x\in[0,1] \\ 0 & , x\notin[0,1] \end{cases}$$

$$F(x) = \begin{cases} 0 & , x<0 \\ \int_0^x f(t)\,dt & , x\in[0,1] \\ 1 & , x>1 \end{cases}$$

pro $x\in[0,1]$: $F(x) = \int_0^x 12t(t-1)^2\,dt = \int_0^x 12t(t^2-2t+1)\,dt$

$= 12\int_0^x (t^3-2t^2+t)\,dt = 12\cdot\left[\frac{t^4}{4} - 2\cdot\frac{t^3}{3} + \frac{t^2}{2}\right]_0^x$

$= 3x^4 - 8x^3 + 6x^2$

$EX = \int_{\mathbb{R}} x f(x)\,dx = \int_0^1 x\cdot 12x(x-1)^2\,dx = 12\int_0^1 (x^4 - 2x^3 + x^2)\,dx$

$= 12\cdot\left[\frac{x^5}{5} - 2\cdot\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1 = 12\cdot\underbrace{\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right)}_{\frac{6-15+10}{30}} = \frac{12}{30} = \underline{\underline{\frac{2}{5}}}$

$EX^2 = \int_{\mathbb{R}} x^2 f(x)\,dx = \int_0^1 x^2\cdot 12x(x-1)^2\,dx = 12\cdot\int_0^1 (x^5 - 2x^4 + x^3)\,dx$

$= 12\cdot\left[\frac{x^6}{6} - 2\cdot\frac{x^5}{5} + \frac{x^4}{4}\right]_0^1 = 12\cdot\underbrace{\left(\frac{1}{6} - \frac{2}{5} + \frac{1}{4}\right)}_{\frac{10-24+15}{60} = \frac{1}{60}} = \frac{12}{60} = \frac{1}{5}$

$\mathrm{var}X = EX^2 - (EX)^2 = \frac{1}{5} - \frac{4}{25} = \underline{\underline{\frac{1}{25}}}$