5. minitest – Matematika M1, 5.11.

$$\lim_{x\to 0}\left(\frac{4-3\cos x}{3-2\cos x}\right)^{\frac{1}{x^2}} = \lim_{x\to 0} e^{\frac{1}{x^2}\cdot \ln\left(\frac{4-3\cos x}{3-2\cos x}\right)} =$$

$$\overset{\text{VOLSF}}{=} e^{\lim_{x\to 0}\frac{1}{x^2}\cdot \ln\left(\frac{4-3\cos x}{3-2\cos x}\right)} \overset{(*)}{=} e^{\frac{1}{2}}$$

$$(*)\ \lim_{x\to 0}\frac{1}{x^2}\cdot \underbrace{\frac{\ln\left(\frac{4-3\cos x}{3-2\cos x}\right)}{\frac{4-3\cos x}{3-2\cos x}-1}}_{\xrightarrow[x\to 0]{\text{VOLSF}}\ 1}\cdot\left(\frac{4-3\cos x}{3-2\cos x}-1\right) =$$

vnitřní funkce $g(x)=\frac{4-3\cos x}{3-2\cos x} \xrightarrow{x\to 0} 1$

vnější funkce $f(y)=\frac{\ln y}{y-1} \xrightarrow{y\to 1} 1$

$$= \lim_{x\to 0}\frac{1}{x^2}\cdot\frac{4-3\cos x-(3-2\cos x)}{3-2\cos x} =$$

$$\overset{\text{VOAL}}{=} \underbrace{\lim_{x\to 0}\frac{1}{3-2\cos x}}_{=1}\cdot\underbrace{\lim_{x\to 0}\frac{1-\cos x}{x^2}}_{=\frac{1}{2}} = \frac{1}{2}$$