

7. minitest - varianta A

Dvojný integrál

22. 11. 2023

Vypočítejte obsah rovinného obrazce omezeného křivkami $y = x - 1$ a $y = x^2 - 6x + 11$.

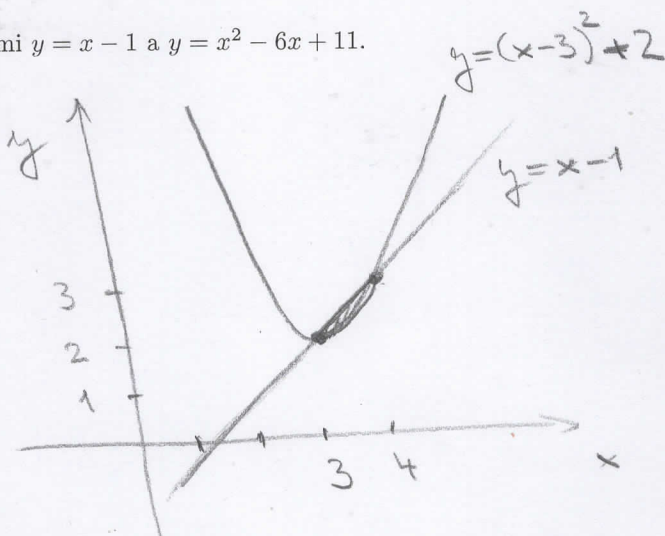
průsečíky: $x - 1 = x^2 - 6x + 11$

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x = 4 \vee x = 3$$

$$y = 3 \quad y = 2$$



$$\int_3^4 \int_{x^2 - 6x + 11}^{x - 1} 1 \, dy \, dx = \int_3^4 (x - 1 - (x^2 - 6x + 11)) \, dx =$$

$$= \int_3^4 (-x^2 + 7x - 12) \, dx = \left[-\frac{x^3}{3} + 7 \cdot \frac{x^2}{2} - 12x \right]_3^4 =$$

$$= -\frac{4^3}{3} + 7 \cdot \frac{4^2}{2} - 12 \cdot 4 - \left(-\frac{3^3}{3} + 7 \cdot \frac{3^2}{2} - 12 \cdot 3 \right) =$$

$$= -\frac{64}{3} + 56 - 48 + 9 - \frac{63}{2} + 36 =$$

$$= 53 - \frac{128 + 189}{6} = \frac{318}{6} - \frac{317}{6} = \underline{\underline{\frac{1}{6}}}$$

7. minitest - varianta B

Dvojný integrál
22. 11. 2023

Vypočítejte obsah rovinného obrazce omezeného křivkami $y = x - 2$ a $y = x^2 - 6x + 10$.

průsečíky:

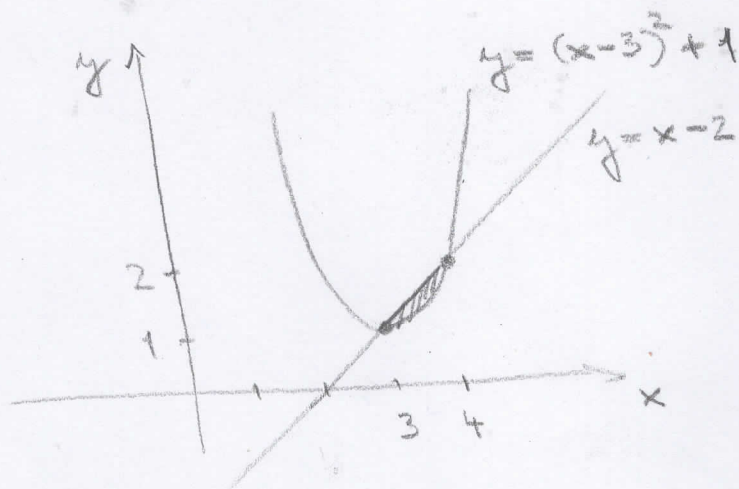
$$x - 2 = x^2 - 6x + 10$$

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x = 4 \quad \vee \quad x = 3$$

$$y = 2 \quad y = 1$$



$$\int_3^4 \int_{x^2 - 6x + 10}^{x - 2} 1 \, dy \, dx = \int_3^4 (x - 2 - (x^2 - 6x + 10)) \, dx =$$

$$= \int_3^4 (-x^2 + 7x - 12) \, dx = \left[-\frac{x^3}{3} + 7 \cdot \frac{x^2}{2} - 12x \right]_3^4 =$$

$$= -\frac{4^3}{3} + 7 \cdot \frac{4^2}{2} - 12 \cdot 4 - \left(-\frac{3^3}{3} + 7 \cdot \frac{3^2}{2} - 12 \cdot 3 \right)$$

$$= -\frac{64}{3} + 56 - 48 + 9 - \frac{63}{3} + 36 = \frac{-128 - 189}{6} + 53$$

$$= \frac{-317 + 318}{6} = \frac{1}{6}$$