

6. minitest - varianta A

Nevlastní integrál

10. 11. 2023

Vypočtete integrál

$$\int_1^{\infty} \frac{1}{x^2 + 3x + 2} dx$$

$$\int \frac{1}{x^2 + 3x + 2} dx = \int \left(\frac{-1}{x+2} + \frac{1}{x+1} \right) dx = -\ln|x+2| + \ln|x+1|$$

$$\frac{1}{x^2 + 3x + 2} = \frac{1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x+2)$$

$$x = -1: 1 = B$$

$$x = -2: 1 = -A \Rightarrow A = -1$$

$$\int_1^{\infty} \frac{1}{x^2 + 3x + 2} dx = \left[\ln \left| \frac{x+1}{x+2} \right| \right]_1^{\infty} = \lim_{x \rightarrow \infty} \ln \left(\frac{x+1}{x+2} \right) - \ln \frac{2}{3}$$

$$= \underbrace{\ln 1}_0 - \ln \frac{2}{3} = \underline{\underline{\ln \frac{3}{2}}}$$

$$\ln \left(\lim_{x \rightarrow \infty} \frac{x+1}{x+2} \right)$$

L'Hôpital

6. minitest - varianta B

Nevlastní integrál
10. 11. 2023

Vypočtěte integrál

$$\int_1^{\infty} \frac{1}{x^2 + 5x + 6} dx$$

$$\int \frac{1}{x^2 + 5x + 6} dx = \int \left(\frac{-1}{x+3} + \frac{1}{x+2} \right) dx =$$

$$= -\ln|x+3| + \ln|x+2|$$

$$\frac{1}{x^2 + 5x + 6} = \frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+3)$$

$$x = -3: 1 = -A \Rightarrow A = -1$$

$$x = -2: 1 = B$$

$$\int_1^{\infty} \frac{1}{x^2 + 5x + 6} dx = \left[\ln \left| \frac{x+2}{x+3} \right| \right]_1^{\infty} = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x+3} \right)$$

$$- \ln \frac{3}{4} = \underbrace{\ln 1}_0 + \ln \frac{4}{3} = \underline{\underline{\ln \frac{4}{3}}} \quad \ln \left(\lim_{x \rightarrow \infty} \frac{x+2}{x+3} \right)$$

L.P. 1