

Übungen B

1) $\sum_{n=1}^{\infty} \left(\frac{2^{n+1} + 3^{n-1} + 3^n}{2^{n-1} + 3^{n+2}} \right)^n$ konvergenz

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \frac{3^n \cdot \left(2 \cdot \left(\frac{2}{3} \right)^n + \frac{1}{3} + 1 \right)}{3^n \cdot \left(\frac{1}{2} \left(\frac{2}{3} \right)^n + 9 \right)} = \frac{\frac{4}{9}}{\frac{27}{9}} = \frac{4}{27} \in [0, 1)$$

$\sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{3^{n+2}} + \frac{n!}{3^n} \right) = \frac{2}{9} \cdot \sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n + \sum_{n=1}^{\infty} \frac{n!}{3^n} = \infty$
 konvergenz divergenz

$\sum_{n=1}^{\infty} (\sqrt{n^2 + n} - n)$ divergenz

$$(\sqrt{n^2 + n} - n) \cdot \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} = \frac{n^1}{n \left(\sqrt{1 + \frac{1}{n}} + 1 \right)} \xrightarrow{n \rightarrow \infty} \frac{1}{2} \neq 0$$

2) $\sum_{n=1}^{\infty} \frac{2^{3n+2} + 2^{3n-2}}{3^{2n-1} + 3^{2n+1}} = \frac{17}{4} \cdot \frac{10}{3} \cdot \sum_{n=1}^{\infty} \left(\frac{8}{9} \right)^n = \frac{17}{4} \cdot \frac{2}{6} \cdot 8 = \frac{51}{5}$
 $\frac{2^{3n} \left(4 + \frac{1}{4} \right)}{3^{2n} \left(\frac{1}{3} + 3 \right)} = \frac{\frac{17}{4} \cdot 8}{1 - \frac{8}{9}} = 8$

3) $\lim_{n \rightarrow \infty} \left(\frac{n+4}{n+3} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+3} \right)^{n+3} \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+3} \right)^{-3} = e \cdot (1+0)^{-3} = 1$

4) $a_{n+2} - a_{n+1} - 12a_n = 0$
 $\lambda^2 - \lambda - 12 = 0$
 $(\lambda - 4)(\lambda + 3) = 0$
 $\lambda = 4 \vee \lambda = -3$

F.S. $4^n, (-3)^n$

$a_n = c_1 \cdot 4^n + c_2 \cdot (-3)^n$
 $a_0 = c_1 + c_2 = 0$ (3)
 $a_1 = 4c_1 - 3c_2 = 1$ (2)

5) $y = \frac{33}{224} e^{4x} - \frac{25}{189} e^{-3x} - \frac{x^2}{12} + \frac{x}{42} - \frac{13}{864}$

$7c_1 = 1$
 $c_1 = \frac{1}{7}$ $c_2 = -\frac{1}{7}$